Universal regular short distance behavior from an interaction with a scale invariant gravity

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Abstract

We assume that the fourdimensional quantum gravity is scale invariant at short distances. We show through a simple scaling argument that correlation functions of quantum fields interacting with gravity have a universal (more regular) short distance behavior.

We consider the tetrads $e^{\mu}_{a}(x)$ as the basic variables for quantum gravity. We assume that correlation functions of $e^{\mu}_{a}(x)$ are scale invariant. This means that (with a certain real γ) $\lambda^{\gamma}e^{\mu}_{a}(\lambda x)$ and $e^{\mu}_{a}(x)$ have the same correlation functions for all real λ,i.e., these are equivalent random variables. Moreover, we assume that at short distances $< e^{\mu}_{a}(x)e^{\nu}_{c}(x') >$ depends only on $x - x'$. The scale invariance cannot apply to intermediate distances where the correspondence with the classical gravity is required. Then, we expect that the mean value of $e^{\mu}_{a}(x)$ describes the classical gravity. We could apply our arguments if the mean value of the tetrad was different from zero and the correlation functions were scale invariant only for short distances. However, for simplicity of the presentation we assume the exact scale invariance. We show that quantum fields interacting with gravity have a more regular behavior than the canonical one. It is determined solely by the scaling index γ. In reality quantum gravity is not expected to be exactly scale invariant. There will be a distance scale (presumably the Planck scale) which determines the region where the scaling applies. There are renormalizable models of gravity (with the square of the curvature tensor) [1][2] [3][4] which may be scale invariant at short distances. Experiments support the canonical behavior of QED at short distances (modified eventually by logarithmic corrections). We suggest that the regular (superrenormalizable) behavior takes over at distances below the Planck scale (there were earlier papers on this topic reviewed by S. Deser in [5], see also [6][7]).

We consider first the free scalar field in four dimensions interacting with gravity. The two-point function in an external gravitational field is (in the

proper time representation)

$$G_g(x,y) \equiv \int_0^\infty d\tau K_\tau(x,y) = \int_0^\infty d\tau \int \mathcal{D}x \exp(-\frac{1}{2}\int g^{\mu\nu}\frac{dx_\mu}{dt}\frac{dx_\nu}{dt}) \\ \delta\left(x\left(0\right)-x\right)\delta\left(x\left(\tau\right)-y\right) \tag{1}$$

We can transform the functional integral (1) into a Gaussian integral by means of a change of variables $x(s) \rightarrow b(s)$. For this purpose we choose $e_a^\mu(x)$ as a square root of the metric (defined up to a rotation)

$$g^{\mu\nu}(x) = e_a^\mu(x)e_a^\nu(x) \tag{2}$$

Then, we define a path $q^\mu(s)$ starting from x and the tetrad e along this path by the system of (Stratonovitch) equations (see [8] or [9][10] for a heuristic derivation)

$$dq^\mu(s) = e_a^\mu\left(q\left(s\right)\right)db^a(s) \tag{3}$$

$$de_a^\mu(q) + \Gamma^\mu_{\nu\rho}(q)e_a^\nu(q)dq^\rho = 0 \tag{4}$$

where Γ is the Christoffel symbol. It follows from eq.(1) that $b^a(s)$ is a Gaussian process (the Brownian motion) with the covariance (the expectation value over the Brownian motion will be denoted by $E[.]$)

$$E[b_a(t)b_c(s)] = \delta_{ac}\min(s,t)$$

When we solve eqs.(3)-(4) then the heat kernel entering eq.(1) is expressed by the formula

$$K_\tau(x,y) = E[\delta(y - q_\tau(x))] \tag{5}$$

The short distance behavior of the two-point function (1) is determined by the behavior of $q(\tau) - x$ for a small τ. We can obtain this behavior from the differential equation (3). Let us assume (here the equivalence is meant in the sense of correlation functions)

$$q(\alpha s) - x \simeq \alpha^\omega(\tilde{q}(s) - x) \tag{6}$$

where $\tilde{q}$ is equivalent to q . Then, using

$$\lambda^\gamma e(\lambda x) \simeq \tilde{e}(x) \tag{7}$$

(where $\tilde{e}(x)$ is a random field equivalent to $e(x)$) and the (approximate) translational invariance of the correlation functions of the tetrads we can determine ω inserting eqs.(6)-(7) into eq.(3)

$$\omega = \frac{1}{2}(1+\gamma)^{-1} \tag{8}$$

Eq.(7) means that $e(x)e(y) \approx |x-y|^{-2\gamma}$ (in the sense of correlation functions). Hence, the singular short distance behavior of the random field e makes the

paths q less regular (we would have $\omega=\frac{1}{2}$ for the Brownian motion as well as for regular tetrads).

In quantum gravity the scalar propagator G results from an average $\langle G_g\rangle$ of G_g in eq.(1) over the gravitational field. In order to calculate this average let us write eq.(3) in an integral form

$$q^\mu(\tau)=x^\mu+\int_0^\tau e_a^\mu\left(q\left(s\right)\right)db^a(s) \tag{9}$$

Inserting the r.h.s. of eq.(9) into eq.(5) we obtain

$$\langle K_\tau(x,y)\rangle=\langle E\left[\delta\left(y-x-\int_0^\tau e_a^\mu\left(q\left(s\right)\right)db^a\left(s\right)\right)\right]\rangle \tag{10}$$

We consider a change of time $s\to\frac{s}{\tau}$ inside the integral (10) and apply the scaling (7) of e, the scaling $b(\alpha s)\simeq\sqrt{\alpha}b(s)$ of the Brownian motion and the scaling (6) of $q(s)$ (with $\alpha=\frac{1}{\tau}$) in order to rewrite the kernel (10) in terms of the paths of the process $\tilde{q}$ defined on the interval $[0,1]$

$$\begin{aligned}&\langle K_\tau(x,y)\rangle=\langle E\left[\delta\left(y-x-\tau^{\frac{1}{2}-\omega\gamma}\int_0^1\tilde{e}_a^\mu\left(\tilde{q}\left(s\right)\right)db^a\left(s\right)\right)\right]\rangle\\&=\tau^{-2+4\omega\gamma}\langle E\left[\delta\left(\left(y-x\right)\tau^{-\frac{1}{2}+\omega\gamma}-\int_0^1\tilde{e}_a^\mu\left(\tilde{q}\left(s\right)\right)db^a\left(s\right)\right)\right]\rangle\end{aligned} \tag{11}$$

Let $F(u)$ be the probability distribution of the random variable

$$\eta^\mu=\int_0^1\tilde{e}_a^\mu\left(\tilde{q}\left(s\right)\right)db^a\left(s\right)$$

Then, using eqs.(8) and (11)

$$\begin{aligned}&G(x,y)=\langle G_g(x,y)\rangle=\int_0^\infty d\tau\tau^{-2+4\gamma\omega}F\left(\left(y-x\right)\tau^{-\frac{1}{2}+\omega\gamma}\right)\\&=2(1+\gamma)|y-x|^{-2+2\gamma}\int_0^\infty dtt^{1-2\gamma}F\left(t\left(y-x\right)|y-x|^{-1}\right)\end{aligned} \tag{12}$$

Strictly speaking correlation functions of the random variable η need renormalization because e has singular correlation functions. This can easily be seen already for the second moment of η. Then, a subtraction of infinities defines a renormalized two-point function of the scalar field (in our earlier paper [11] we have discussed such a renormalization in a particular model with a Gaussian tetrad). However, the result (12) does not depend on the way we renormalize η. It follows just from the scaling of τ. The behavior of G is more regular if the gravitational correlations are more singular (in [11] we obtained a bound $\gamma<\frac{1}{2}$ resulting from the requirement of renormalizability).

We could treat the perturbative expansion of ϕ^4 by means of the functional representation (1). In such a case we can show just by the scaling argument that the correlations of the fourth power of ϕ have the singularity of the fourth power of the two point function (12). The model becomes superrenormalizable. We

can repeat the argument for other fields entering the Standard Model. We have the path integral representation of the two-point function in the gravitational field analogous to eq.(1) (we consider the square of the Dirac operator is such a representation). There are factors multiplying the δ-function inside the path integral (5). However, such factors have no effect on the scaling argument. Hence, the estimate (12) remains true for the propagator of the gauge fields as well as for the propagator of the square of the Dirac operator.

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